

Automated Validation of Security-sensitive Web Services specified in BPEL and RBAC

(Extended Version)

Alberto Calvi
Dipartimento di Informatica
Università di Verona, Italy
Email: alberto.calvi@univr.it

Silvio Ranise
FBK-Irst (Trento, Italy)
Email: ranise@fbk.eu

Luca Viganò
Dipartimento di Informatica
Università di Verona, Italy
Email: luca.vigano@univr.it

Abstract—We formalize automated analysis techniques for the validation of web services specified in BPEL and a RBAC variant tailored to BPEL. The idea is to use decidable fragments of first-order logic to describe the state space of a certain class of web services and then use state-of-the-art SMT solvers to handle their reachability problems. To assess the practical viability of our approach, we have developed a prototype tool implementing our techniques and applied it to a digital contract signing service inspired by an industrial case study.

I. INTRODUCTION

Context and motivation: The design of security-sensitive web services is an error-prone and time-consuming task. The reasons of these difficulties are manifold. A web service is (often) obtained as a composition of several simpler services executed in a distributed environment. So, because of the huge number of possible interleavings and the subtle interplay between the data and the control part of the processes, it is very difficult—if not impossible—for a human to foresee all the possible behaviors. Furthermore, the workflow of the application is usually constrained by the enforcement of access-control policies that forbid the execution of certain operations or the access to shared resources by certain users and can easily over-constrain or under-constrain the possible behaviors. As a consequence, in the first case, correct behaviors are prevented, thereby decreasing the overall dependability of the service, while in the second case, incorrect behaviors are possible that may open security breaches, thus destroying the dependability of the service.

Hence, automated techniques for the validation of security-sensitive web services at design time are needed to assist the designers and avoid expensive actions for the correction of errors after deployment. While this is only a preliminary step in the direction of building highly dependable web services, it constitutes a necessary stepping stone for the application of other techniques at run-time for orchestration and coordination of services and for enforcing access policies.

Contributions: In this paper, we formalize automated analysis techniques for the validation of web services specified in BPEL and a variant of RBAC tailored to BPEL as proposed in [1]. RBAC (see, e.g., [2]) is one of the most successful

models for access control in large and complex applications. Our idea is to translate a BPEL process to a particular class of transition systems, described by arithmetic constraints and called Vector Addition Systems (VASs), and to encode the RBAC specification in a decidable class of first-order formulae, called Bernays-Schönfinkel-Ramsey (BSR). We study the goal reachability problem (to which several analysis problems can be reduced) of the resulting class of transition systems. Theoretically, we prove the decidability of the reachability problem for a particular class of transition systems modeling BPEL processes where no loops occur. Pragmatically, to assess the viability of our approach, we have developed a prototype tool called WSSMT, which implements our techniques and uses state-of-the-art theorem-proving techniques recently developed in the area of Satisfiability Modulo Theories (SMT) and featuring a good trade-off between scalability and expressiveness. We report on the application of WSSMT on a digital contract signing service inspired by an industrial case study.

Related work: While BPEL semantics is given in natural language in [3], there have been many attempts to give a formal semantics of the language in terms of Petri nets, e.g., [4]. The formalization is useful in two respects: it eliminates possible ambiguities in the natural language semantics and it permits the formal analysis of BPEL processes at the design time. Although there are tools (e.g., [5]) that provide automated support for the translation from BPEL to Petri nets and the subsequent analysis, they (to the best of our knowledge) only model the control flow and abstract away from the data manipulation. Recently, there have been attempts at extending Petri nets with some data modeling and reasoning capabilities by using fragments of first-order logic (FOL) for which efficient SMT solvers exist [6]. Instead of a hybrid representation, we chose to develop our techniques in a first-order framework by exploiting the well-known connection between Petri nets and VASs (see, e.g., [7], [8]) and then to extend it along the lines suggested in [9] to incorporate the access-control layer in a uniform way. The work in [9] studies the decidability of symbolic executions with bounded length for more general classes of services while here we focus on a particular class of applications whose sets of reachable states

can be finitely described by suitable fragments of FOL. The work in [10] is closely related to ours with respect to the structure of the specification divided in two layers, one for the workflow and one for access control. However, [10] does not provide a decidability result for the reachability problem as we do in this paper.

Organization: § II briefly introduces the languages used to specify the class of web services we consider, i.e. BPEL and RBAC4BPEL, together with a concrete example that we use to illustrate the key features of the formal framework. § III recalls the definition of two-level transition system introduced in [9], and its related reachability problem, explains how BPEL and RBAC4BPEL specifications can be translated to this class of transition systems, and proves the decidability of the reachability problem for two-level transition systems obtained by translating a class of acyclic Petri nets (called workflow nets). § IV discusses how our techniques have been implemented and applied to a digital contract signing service, inspired by an industrial application. In § V, we draw conclusions and discuss future work. Proofs of the formal results are given in an appendix.

II. BPEL, RBAC4BPEL, AND A MOTIVATING EXAMPLE

We characterize the class of applications we are interested in by using the Purchase Ordering (PO) process introduced in [1]. To make the paper self contained, in this section, we briefly illustrate the example and give a high-level description of the languages used to specify it. The PO process is composed of six activities: the creation of a purchase order for a certain good (crtPO), the approval of the order before dispatching the good to the supplier (apprPO), the acknowledgement of the delivery by signing (signGRN) and then countersigning (ctr-signGRN) the goods-received note, the creation of a payment file on receipt of the supplier's invoice for the good (crtPay), and the approval of the payment to the supplier (apprPay). For the PO process to complete successfully, the order of execution of the various activities should satisfy the following constraints: crtPO must be executed before apprPO which, in turn, must be executed before the remaining four activities; crtPay can be done in parallel with both signGRN and ctr-signGRN but before apprPay; and signGRN, ctrsignGRN, and apprPay must be executed in this order. The *workflow (WF) level* of the application should enforce these dependencies that are induced by the application logic of the PO process.

A. The WF level and BPEL

In [1], the Business Process Execution Language (BPEL [3]) is used to describe a (executable) specification of the WF level of an application. In Fig. 1, we show a high-level BPEL specification of the WF level of the PO process. The `<process>` element wraps around the entire description of the PO process. The `<sequence>` element states that the activities contained in its scope must be executed sequentially. The `<flow>` element specifies concurrent threads of activities. The `<invoke>` element represents the invocation of an activity that is provided by an available web service.

```
<process name="PO"/>
<sequence>
  <receive operation="crtPO" ... > </receive>
  <invoke operation="apprPO" ... > </invoke>
  <flow>
    <sequence>
      <invoke operation="signGRN" ... > </invoke>
      <invoke operation="ctrsignGRN" ... > </invoke>
    </sequence>
    <invoke operation="ctrPay" ... > </invoke>
  </flow>
  <invoke operation="apprPay" ... > </invoke>
</sequence>
</process>
```

Fig. 1. The WF level of the PO process: BPEL.

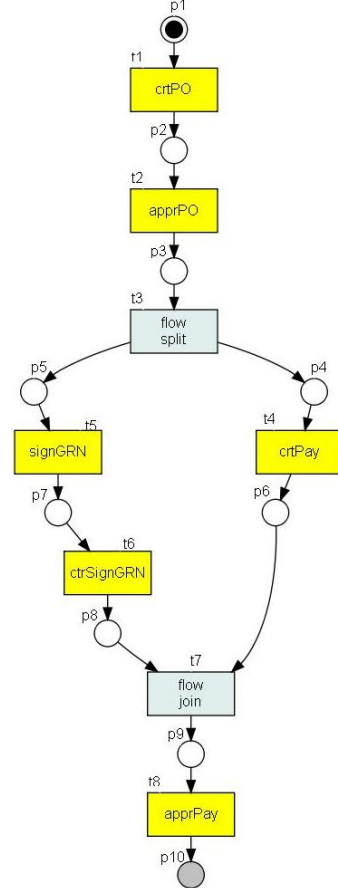


Fig. 2. The WF level of the PO process: Petri Net corresponding to BPEL in Fig. 1.

Finally, the `<receive>` element represents the invocation of an activity that is provided by the BPEL process being described. Indeed, BPEL provides a variety of constructs (e.g., to represent variables) that are ignored here for simplicity; the interested reader is pointed to [3]. In the case of the PO process, it is easy to see that the constraints on the execution described above are all satisfied by the nesting of control elements in Fig. 1. For example, because of the semantics of `<sequence>`, crtPO will be executed first while apprPay will be the activity finishing the PO process.

Fig. 2 shows a Petri net that can be seen as the (formal)

$$\begin{aligned}
U &:= \{u_1, u_2, u_3, u_4, u_5\} & R &:= \{Manager, FinAdmin, FinClerk, POAdmin, POCLerk\} \\
P &:= \{p_1, \dots, p_5\} & ua &:= \{(u_1, Manager), (u_2, FinAdmin), (u_3, FinClerk), (u_4, POAdmin), (u_5, POCLerk)\} \\
pa &:= \{(FinClerk, p_4), (FinAdmin, p_5), (POCLerk, p_3), (POAdmin, p_1)\} \\
&\succeq \text{ least partial order s.t. } Manager \succeq FinAdmin, Manager \succeq POAdmin, \\
&\quad FinAdmin \succeq FinClerk, \text{ and } POAdmin \succeq POCLerk.
\end{aligned}$$

Fig. 3. The PM level of the PO process: RBAC

semantics of the BPEL process in Fig. 1. Before being able to sketch the mapping from BPEL processes to Petri nets, we recall the basic notions concerning the latter.

A *Petri net* is a triple $\langle P, T, F \rangle$, where P is a finite set of *places*, T is a finite set of *transitions*, and F (*flow relation*) is a set of arcs such that $P \cap T = \emptyset$ and $F \subseteq (P \times T) \cup (T \times P)$. Graphically, the Petri net $\langle P, T, F \rangle$ can be depicted as a directed bipartite graph with two types of nodes, places and transitions, represented by circles and rectangles, respectively; the nodes are connected via directed arcs according to F (where arcs between two nodes of the same type are not allowed). A place p is called an *input* (resp., *output*) place of a transition t iff there exists a directed arc from p to t (resp., from t to p). The set of input (resp., output) places of a transition t is denoted by $\bullet t$ (resp., $t\bullet$); $\bullet p$ and $p\bullet$ are defined similarly. A *path* in a Petri net $\langle P, T, F \rangle$ is a finite sequence e_0, \dots, e_n of elements from $P \cup T$ such that $e_{i+1} \in e_i\bullet$ for each $i = 0, \dots, n-1$; a path e_0, \dots, e_n in the net is a *cycle* if no element occurs more than once in it and $e_0 \in e_n\bullet$ for some $n \geq 1$. A Petri net is *acyclic* if none of its paths is a cycle. A *marking* of a Petri net $\langle P, T, F \rangle$ is a mapping from the set P of places to the set of non-negative integers; graphically, it is depicted as a distribution of black dots in the circles of the graph representing the net. A transition t is *enabled* in a marking m iff each of its input places p is such that $m(p) \geq 1$, i.e., p contains at least one token. An enabled transition t in a marking m may *fire* by generating a new marking m' , in symbols $m \xrightarrow{t} m'$, where $m'(p) = m(p)$ if $p \notin (\bullet t \cup t\bullet)$, $m'(p) = m(p) - 1$ if $p \in \bullet t$, and $m'(p) = m(p) + 1$ if $p \in t\bullet$, i.e. t consumes one token from each input place of t and produces one token in each of its output places. A marking m is *reachable* from m_0 , in symbols $m_0 \rightarrow^* m$, iff there exists a sequence m_1, \dots, m_n of markings such that $m_i \xrightarrow{t} m_{i+1}$ for $i = 0, \dots, n-1$ and $m_n = m$, for some $n \geq 0$. (In case $n = 0$, we have that $m_0 = m$.) Given a Petri net $\langle P, T, F \rangle$ and a marking m , an instance of the *reachable problem* for Petri nets consists of checking whether $m_0 \rightarrow^* m$ or not. A *workflow (WF) net* [11] is a Petri net $\langle P, T, F \rangle$ such that (a) there exist two special places $i, o \in P$ with $\bullet i = \emptyset$ and $o\bullet = \emptyset$; and (b) for each transition $t \in T$, there exists a path π in the net beginning with i and ending with o in which t occurs.

The idea underlying the Petri net semantics of BPEL is simple. Activities are mapped to transitions (the rectangles in Fig. 2) and their execution is modeled by the flow of tokens from input places to output places. When two BPEL operations are enclosed in a *<sequence>* element (e.g.,

crtpo and apprpo), two transitions are created (as in Fig. 2) with one input place (resp., p_1 and p_2) and one output place each (resp., p_2 and p_3), and the input place of the second is identified with the output place of the first one (p_2). When two BPEL operations are in a *<flow>* element (e.g., ctrpay and the sequence of operations signGRN and ctrsignGRN), four transitions are created: one to represent the split of the flow, one to represent its synchronization (join), and one for each activity that can be executed concurrently with the appropriate places to connect them (in Fig. 2, when a token is in place p_3 , the ‘flow split’ transition is enabled and its execution yields one token in place p_4 and one in p_5 , which enables both transitions signGRN and ctrpay that can be executed concurrently; the two independent threads of activities get synchronized again by the execution of ‘flow join’, which is enabled when a token is in p_6 and a token is in p_8). It is easy to see that the Petri net of Fig. 2 is an acyclic WF net where p_1 is the special input place i , p_{10} is the special output place o , and each transition occurs in a path from p_1 to p_{10} .

B. The policy management (PM) level and RBAC4BPEL

Besides the dependencies imposed by the WF level, constraints on the execution of the activities derived from security requirements are of crucial importance to ensure the dependability of the application. In this paper, we focus on a particular class of security requirements that pertain to the access-control policy. The *policy management (PM) level* of the application is charged to enforce these constraints.

In [1], an extension of the Role Based Access Control (RBAC) framework—adapted to work smoothly with BPEL, denoted with RBAC4BPEL—is used to specify the PM level of applications. The components of RBAC4BPEL are: (i) a set U of users, (ii) a set R of roles, (iii) a set P of permissions, (iv) a role hierarchy \succeq (i.e. a partial-order relation on R), (v) a user-role assignment relation ua , (vi) a role-permission assignment relation pa , and (vii) a set A of activities and a class of authorization constraints (such as separation-of-duty) to prevent some user to acquire permissions in certain executions of the application (see below for details). Note that components (i)–(vi) are standard in RBAC while (vii) has been added to obtain a better integration between the PM and the WF levels.

First, we describe components (i)–(vi) and some related notions. A user $u \in U$ is assigned a role $r \in R$ if $(u, r) \in ua$ and permissions are associated with roles when $(p, r) \in pa$. In RBAC4BPEL, a user $u \in U$ has a permission p if there exists a role $r \in R$ such that $(u, r) \in ua$ and $(p, r) \in pa$. (We will

see that each permission is associated to a right on a certain activity in A —e.g., its execution—of a BPEL process.) The role hierarchy $\succeq \subseteq R \times R$ is assumed to be a partial order (i.e., a reflexive, antisymmetric, and transitive relation) reflecting the rights associated to roles. More precisely, a user u is an *explicit* member of role $r \in R$ if $(u, r) \in ua$ and it is an *implicit* member of role $r \in R$ if there exists a role $r' \in R$ such that $(r', r) \in \succeq$ (abbreviated as $r' \succeq r$), $r' \neq r$, and $(u, r') \in ua$. Thus, \succeq induces a permission inheritance relation as follows: a user $u \in U$ can get permission p if there exists a role $r \in R$ such that u is a member (either implicit or explicit) of r and $(p, r) \in pa$. For simplicity, we abstract away the definition of a role in terms of a set of attributes as done in [1].

Fig. 3 shows the sets U, R, P and the relations \succeq, ua, pa for the PM level of the PO process. Although $(Manager, p_i) \notin pa$ for any $i = 1, \dots, 5$, we have that user u_1 , which is explicitly assigned to role *Manager* in ua , can get any permission p_i for $i = 2, \dots, 5$ as $Manager \succeq r$ for any role $r \in R \setminus \{Manager\}$, hence u_1 can be implicitly assigned to each role and then get the permission p_i .

In RBAC4BPEL, each permission in P is associated with the right to handle a certain transition of T , uniquely identified by a label in A , for a Petri net $\langle P, T, F \rangle$. For the PO process, this is particularly simple since only the right to execute a transition is considered. So, p_1 is the permission for executing *apprPO*, p_2 for *signGNR*, p_3 for *ctrSignGNR*, p_4 for *crtPay*, and p_5 for *apprPay*. We are now in the position to describe component (vii) of RBAC4BPEL. Note that there are no permissions associated to ‘flow split’ and ‘flow join’ as these are performed by the BPEL engine and thus no particular authorization restriction must be enforced.

A *role* (resp., *user*) *authorization constraint* is a tuple $\langle D, (t_1, t_2), \rho \rangle$ if $D \subseteq R$ (resp., $D \subseteq U$) is the domain of the constraint, $\rho \subseteq R \times R$ (resp., $\rho \subseteq U \times U$), and t_1, t_2 are in A . An authorization constraint $\langle D, (t_1, t_2), \rho \rangle$ is *satisfied* if $(x, y) \in \rho$ when $x, y \in D$, x performs t_1 , and y performs t_2 . In other words, authorization constraints place further restrictions (besides those of the standard RBAC components) on the roles or users who can perform certain actions once others have been already executed by users belonging to certain roles. Constraints of this kind allow one to specify separation-of-duty (SoD) by $\langle D, (t_1, t_2), \neq \rangle$, binding-of-duty (BoD) by $\langle D, (t_1, t_2), = \rangle$, or any other restrictions that can be specified by a binary relation over roles or users.

For the PO process, (vii) of RBAC4BPEL is instantiated as: $\langle U, (apprPO, signGNR), \neq \rangle$, $\langle U, (apprPO, crtSignGNR), \neq \rangle$, $\langle U, (signGNR, crtSignGNR), \neq \rangle$, $\langle R, (crtPay, apprPay), \prec \rangle$,

where $\prec := \{(r_1, r_2) \mid r_1, r_2 \in R, r_2 \succeq r_1, r_1 \neq r_2\}$ (recall that the sets U and R are defined in Fig. 3).

This concludes the description of the class of applications that we consider. We now proceed to introduce our techniques to analyze such applications.

III. FORMALIZATION AND AUTOMATED ANALYSIS

From now on, we assume that the WF level of an application is specified by a Petri net and the PM level by an instance

of the RBAC4BPEL framework. We use two-level transition systems [9] to represent the WF level and the PM level of a web-service and we study the reachability problem for a sub-class.

A. Two-level transition systems and goal reachability

We assume the basic notions of FOL (see, e.g., [12]). A *two-level transition system* Tr is a tuple

$$\langle \underline{x}, \underline{p}, In(\underline{x}, \underline{p}), \{\tau_i(\underline{x}, \underline{p}, \underline{x}', \underline{p}') \mid i = 1, \dots, n\} \rangle,$$

where \underline{x} is a tuple of WF state variables, \underline{p} is a tuple of PM state variables, the *initial condition* $In(\underline{x}, \underline{p})$ is a FOL formula whose only free variables are in \underline{x} and where PM state variables in \underline{p} may occur as predicate symbols, the *transition* $\tau_i(\underline{x}, \underline{p}, \underline{x}', \underline{p}')$ is a FOL formula whose only free variables are in $\underline{x}, \underline{x}'$ and where PM state variables in $\underline{p}, \underline{p}'$ may occur as predicate symbols (as it is customary, unprimed variables in τ_i refer to the values of the state before the execution of the transition while those primed to the values of the state afterward) for $i = 1, \dots, n$ and $n \geq 1$.

We assume there exists a so-called first-order *underlying structure* $\langle D, I \rangle$ of the transition system Tr , where D is the domain of values and I is the mapping from the signature to functions and relations over D , and in which the state variables and the symbols of the signature used to write the formulae In and τ_i for $i = 1, \dots, n$ are mapped. A *state* of Tr is a pair $v := (v_{\underline{x}}, v_{\underline{p}})$ of mappings: $v_{\underline{x}}$ from the WF state variables to D and $v_{\underline{p}}$ from the PM state variables to relations over D . A *run* of Tr is a (possibly infinite) sequence of states $v^0, v^1, \dots, v_n, \dots$ such that (a) v^0 satisfies In , in symbols $v^0 \models In$, and (b) for every pair v^i, v^{i+1} in the sequence, there exists $j \in \{1, \dots, n\}$ such that v^i, v^{i+1} satisfies τ_j , in symbols $v^i, v^{i+1} \models \tau_j$, where the domain of v^i is $\underline{x}, \underline{p}$ and that of v^{i+1} is $\underline{x}', \underline{p}'$. Given a formula $G(\underline{x}, \underline{p})$, called the *goal*, an instance of the *goal reachability problem* for Tr consists of answering the following question: does there exist a natural number $\ell \geq 0$ such that the formula

$$In(\underline{x}_0, \underline{p}_0) \wedge \bigwedge_{i=0}^{\ell-1} \tau(\underline{x}_i, \underline{p}_i, \underline{x}_{i+1}, \underline{p}_{i+1}) \wedge G(\underline{x}_\ell, \underline{p}_\ell) \quad (1)$$

is satisfiable in the underlying structure of Tr , where $\underline{x}_i, \underline{p}_i$ are renamed copies of the state variables in $\underline{x}, \underline{p}$? (When $\ell = 0$, (1) is simply $In(\underline{x}_0, \underline{p}_0) \wedge G(\underline{x}_0, \underline{p}_0)$.) The interest of the goal reachability problem lies in the fact that many verification problems for two-level transition systems, such as deadlock freedom and invariant checking, can be reduced to it.

B. Forward reachability and symbolic execution tree

If we were able to check automatically the satisfiability of (1), an idea to solve the goal reachability problem for two-level transition systems would be to generate instances of (1) for increasing values of ℓ . However, this would only give us a semi-decision procedure for the reachability problem. In fact, this method terminates only when the goal is reachable from the initial state, i.e. when the instance of (1) for a certain value of ℓ is unsatisfiable in the underlying structure of the transition system Tr . But, when the goal is not reachable, the check will never detect the unsatisfiability and we will

be bound to generating an infinite sequence of instances of (1) for increasing values of ℓ . That is, the decidability of the satisfiability of (1) in the underlying structure of Tr is only a necessary condition for ensuring the decidability of the goal reachability problem.

We can formalize this method as follows. The *post-image* of a formula $K(\underline{x}, \underline{p})$ with respect to a transition τ_i is

$$Post(K, \tau_i) := \exists \underline{x}', \underline{p}'. (K(\underline{x}', \underline{p}') \wedge \tau_i(\underline{x}', \underline{p}', \underline{x}, \underline{p})).$$

For the class of transition systems that we consider below, we are always able to find FOL formulae that are equivalent to $Post(K, \tau_i)$. Thus, the use of the second-order quantifier over the predicate symbols in \underline{p}' should not worry the reader (see §III-D for details). Now, define the following sequence of formulae by recursion: $FR^0(K, \tau) := K$ and $FR^{i+1}(K, \tau) := Post^i(FR^i, \tau) \vee FR^i(K, \tau)$, for $i \geq 0$ and $\tau := \bigvee_{k=1}^n \tau_k$. The formula $FR^\ell(K, In)$ describes the set of states of the transition system Tr that are *forward reachable in $\ell \geq 0$ steps*. A *fix-point* is the least value of ℓ such that $FR^{\ell+1}(\tau, In) \Rightarrow FR^\ell(\tau, In)$ is true in the structure underlying Tr . Note also that $FR^\ell(\tau, In) \Rightarrow FR^{\ell+1}(\tau, In)$ by construction and hence if $FR^{\ell+1}(\tau, In) \Rightarrow FR^\ell(\tau, In)$ is valid, then also $FR^\ell(\tau, In) \Leftrightarrow FR^{\ell+1}(\tau, In)$ is so and $FR^\ell(\tau, In) \Leftrightarrow FR^{\ell'}(\tau, In)$ for each $\ell' \geq \ell$. Using the sequence of formulae $FR^0(\tau, In), FR^1(\tau, In), \dots$ it is possible to check if the goal property G will be reached by checking whether $FR^\ell(\tau, In) \wedge G$ is satisfiable in the structure underlying Tr for some $\ell \geq 0$. In case of satisfiability, we say that G is *reachable*. Otherwise, if $FR^\ell(\tau, In)$ is a fix-point, the unsatisfiability of $FR^\ell(\tau, In) \wedge G$ implies that G is *unreachable*. Finally, if $FR^\ell(\tau, In)$ is not a fix-point and $FR^\ell(\tau, In) \wedge G$ is unsatisfiable, then we must increase the value of ℓ by 1 so as to compute the set of forward reachable states in $\ell + 1$ steps and perform the reachability checks again. Unfortunately, also this process is not guaranteed to terminate for arbitrary two-level transition systems. Fortunately, we are able to characterize a set of transition systems, corresponding to a relevant class of applications specified in BPEL and RBAC4BPEL, for which we can pre-compute an upper bound on ℓ ; this paves the way to solving automatically the goal reachability problem for these systems. To this end, we consider three sufficient conditions to automate the solution of the goal reachability problem. First, the class \mathcal{C} of formulae used to describe sets of states must be closed under post-image computation. Second, the satisfiability (in the structure underlying the transition system) of \mathcal{C} must be decidable. Third, it must be possible to pre-compute a bound on the length of the sequence $FR^0, FR^1, \dots, FR^\ell$ of formulae. Below, we show that these conditions are satisfied by a class of two-level transition systems to which applications specified in BPEL and RBAC4BPEL can be mapped. For ease of exposition, we first consider the WF and PM levels in isolation and then show how the results for each level can be modularly lifted when considering the two levels together. Before doing this, we introduce the notion of ‘symbolic execution tree.’ The purpose of this is two-fold. First, it is crucial for the technical

development of our decidability result. Second, it is the starting point for the implementation of our techniques as discussed in §IV.

The *symbolic execution tree of the two-level transition system Tr* is a labeled tree defined as follows: (i) the root node is labeled by the formula In , (ii) a node n labeled by the formula K has $d \leq n$ sons n_1, \dots, n_d labeled by the formulae $Post(\tau_1, K), \dots, Post(\tau_d, K)$ such that $Post(\tau_j, K)$ is satisfiable in the model underlying Tr and the edge from n to n_j is labeled by τ_j for $j = 1, \dots, d$, (iii) a node n labeled by K has no son, in which case n is a *final node*, if $Post(\tau_j, K)$ is unsatisfiable in the underlying model of the VAS, for each $j = 1, \dots, n$. A symbolic execution tree is *0-complete* if it consists of the root node labeled by the formula In , it is *(d + 1)-complete* for $d \geq 0$ if its depth is $d + 1$ and for each node n labeled by a formula K_n at depth d , if $Post(\tau_j, K_n)$ is satisfiable, then there exists a node n' at depth $d + 1$ labeled by $Post(\tau_j, K_n)$. In other words, a symbolic execution tree is *d-complete* when all non-empty sets of forward states reachable in one step represented by formulae labeling nodes at depth $d - 1$ have been generated. It is easy to see that the formula $FR^\ell(K, In)$, describing the set of states of the transition system Tr forward reachable in $\ell \geq 0$ steps, is equivalent to the disjunction of the formulae labeling the nodes of an ℓ -complete symbolic execution tree. This will be proved for the classes of two-level transition systems that we consider below.

C. WF nets and terminating forward reachability

We consider a particular class of two-level transition systems, called *Vector Addition System (VAS)*, $\langle \underline{x}, In(\underline{x}), \{\tau_i(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$, such that (a) $\underline{p} = \emptyset$; (b) their underlying structure is that of integers; (c) each WF state variable in $\underline{x} = x_1, \dots, x_m$ ranges over the set of non-negative integers; (d) the initial condition $In(\underline{x})$ is a formula of the form $x_i \bowtie c_1 \wedge \dots \wedge x_m \bowtie c_m$, where c_j is a natural number for $j = 1, \dots, m$ and $\bowtie \in \{=, \neq, >, \geq\}$; and (e) each transition τ_i , for $i = 1, \dots, n$, is a formula of the form

$$\bigwedge_{i \in P} x_i \geq 0 \wedge \bigwedge_{j \in U^+} x'_j = x_j + 1 \wedge \bigwedge_{k \in U^-} x'_k = x_k - 1 \wedge \bigwedge_{l \in U^=} x'_l = x_l,$$

where $P, U^+, U^-, U^=$ are subsets of $\{1, \dots, n\}$ such that $U^+, U^-, U^=$ form a partition of $\{1, \dots, n\}$.

It is well-known that Petri nets and VASs are equivalent in the sense that analysis problems for the former can be transformed to problems of the latter whose solutions can be mapped back to solutions for the original problem and vice versa (see, e.g., [8]). We briefly describe the correspondence by considering the Petri net in Fig. 2. We associate an integer variable x_i to each place p_i for $i = 1, \dots, 10$ whose value will be the number of tokens in the place. The state is given by the value of the integer variables that represents the marking of the net, i.e. a mapping from the set of places to non-negative integers. Formulae can be used to represent sets of states (or, equivalently, of markings). So, for example, the formula $x_1 = 1 \wedge \bigwedge_{i=2}^{10} x_i = 0$ represents the marking where one token is in place p_1 and all the other places are empty (which is the

one depicted in Fig. 2 where the token is represented by a solid circle inside that represents the place p_1 while all the other places do not contain any solid circle). The transition crtPO is represented by the formula

$$x_1 \geq 1 \wedge x'_1 = x_1 - 1 \wedge x'_2 = x_2 + 1 \wedge \bigwedge_{i=3}^{10} x_i = x_i$$

saying that it is enabled when there is at least one token in p_1 ($x_1 \geq 1$) and the result of its execution is that a token is consumed at place p_1 ($x'_1 = x_1 - 1$), the tokens in p_2 are incremented by one ($x'_2 = x_2 + 1$), while the tokens in all the other places are unaffected ($x'_i = x_i$ for $i = 3, \dots, 10$). The other transitions of the Petri net in Fig. 2 are translated in a similar way. In general, it is always possible to associate a state of a VAS to a marking of a Petri net and vice versa. This implies that solving the reachability problem for a VAS is equivalent to solving the reachability problem of the associated Petri net.

Now, we show that the three sufficient conditions (see § III-B) to mechanize the solution of the goal reachability problem are satisfied by VASs when using forward reachability. First, the class of formulae is closed under post-image computation.

Fact 1: $Post(K, \tau_i)$ is equivalent to $K[x_j + 1, x_k - 1, x_l] \wedge \bigwedge_{i \in P} x_i \geq 0$, where $K[x_j + 1, x_k - 1, x_l]$ denotes the formula obtained by replacing x'_j with $x_j - 1$ for $j \in U^+$, x'_k with $x_k - 1$ for $k \in U^-$, and x'_l with $x_l + 1$ for $j \in U^-$. ■

As a corollary, it is immediate to derive that if K is a formula of Linear Arithmetic (LA) [13]—roughly, a formula where multiplication between variables is forbidden—then also $Post(K, \tau_i)$ is equivalent to an effectively computable formula of LA. Second, the satisfiability of the class of formulae of LA is decidable by well-known results [13]. Third, it is possible to pre-compute a bound on the length of the sequence $FR^0, FR^1, \dots, FR^\ell$ of formulae. Using the notion of symbolic execution tree introduced above, once specialized to VASs, we can then prove:

Lemma 1: Let $PN := \langle P, T, F \rangle$ be an acyclic workflow net and Π be the set of all its paths. Then, the set of forward reachable states of the VAS $\langle \underline{x}, In(\underline{x}), \{\tau_i(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$ associated to PN is identified by the formula $FR^\ell(\tau, In)$ for $\ell = \max_{\pi \in \Pi} \{len(\pi|_T)\}$, where $\pi|_T$ is the sequence obtained from π by forgetting each of its elements in P and $len(\pi|_T)$ is the length of the sequence $\pi|_T$. ■

D. RBAC4BPEL and terminating forward reachability

Preliminarily, let $Enum(\{v_1, \dots, v_n\}, S)$ be the following set of FOL formulae axiomatizing the enumerated datatype with values v_1, \dots, v_n for a given $n \geq 1$ over a type S : $v_i \neq v_j$ for each pair (i, j) of numbers in $\{1, \dots, n\}$ such that $i \neq j$ and $\forall x. (x = v_1 \vee \dots \vee x = v_n)$, where x is a variable of type S . The formulae in $Enum(\{v_1, \dots, v_n\}, S)$ fix the number of elements of any interpretation to be v_1, \dots, v_n ; it is easy to see that the class of structures satisfying these formulae are closed under isomorphism. We consider a particular class of two-level transition systems, called RBAC4BPEL,

$\langle \underline{p}, In(\underline{p}), \{\tau_i(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$ such that (a) $\underline{x} = \emptyset$; (b) the initial condition $In(\underline{p})$ is of the form $\forall \underline{w}. \varphi(\underline{w})$, where φ is a quantifier-free formula where at most the variables in \underline{w} may occur free; and (c) the underlying structure is one in the (isomorphic) class of many-sorted structures axiomatized by the following sentences:

$$\begin{aligned} & Enum(U, User), Enum(R, Role), \\ & Enum(P, Permission), Enum(A, Action), \\ & \forall u, r. (ua(u, r) \Leftrightarrow \bigvee_{c_u \in U_{ua}, c_r \in R_{ua}} (u = c_u \wedge r = c_r)) \\ & \forall r, p. (pa(r, p) \Leftrightarrow \bigvee_{c_r \in R_{pa}, c_p \in P_{pa}} (r = c_r \wedge p = c_p)) \\ & c_r \succeq c'_r \text{ for } c_r, c'_r \in R \\ & \forall r. (r \succeq r) \quad \forall r_1, r_2, r_3. (r_1 \succeq r_2 \wedge r_2 \succeq r_3 \Rightarrow r_1 \succeq r_3) \\ & \forall r_1, r_2. (r_1 \succeq r_2 \wedge r_2 \succeq r_1 \Rightarrow r_1 = r_2), \end{aligned}$$

where U , R and P are finite sets of constants denoting users, roles, and permissions, respectively, A is a finite set of actions, u is a variable of type *User*, r and its subscripted versions are variables of type *Role*, p is a variable of type *Permission*, $U_{ua} \subseteq U$, $R_{ua} \subseteq R$, $R_{pa} \subseteq R$, and $P_{pa} \subseteq P$; (d) $p = xcd$ is a predicate symbol of type *User* \times *Action* abbreviating *executed*; and (e) each τ_i is of the form

$$\exists \underline{u}. (\xi(\underline{u}, xcd) \wedge \forall x, y. (xcd'(x, y) \Leftrightarrow ((x = u_j \wedge y = p) \vee xcd(x, y)))),$$

where \underline{u} is a tuple of existentially quantified variables of type *User*, u_j is the variable at position j in \underline{u} , and $\xi(\underline{u})$ is a quantifier-free formula (called the *guard* of the transition) where no function symbol of arity greater than 0 may occur (the part of τ_i specifying xcd' is called the *update*).

We now explain how an RBAC4BPEL system can be specified by the formulae above on the example described in § II. To constrain the sets of users, of roles, and permissions to contain exactly the elements specified in Fig. 3, it is sufficient to use the following sets of formulae: $Enum(\{u_1, u_2, u_3, u_4\}, User)$, $Enum(\{Manager, FinAdmin, FinClerk, POAdmin, PO-Clerk\}, Role)$, and $Enum(\{p_1, p_2, p_3, p_4, p_5\}, Permission)$. It is also easy to see that the formulae

$$\begin{aligned} \forall u, r. (ua(u, r) \Leftrightarrow & \left(\begin{array}{l} (u = u_1 \wedge r = Manager) \vee \\ (u = u_2 \wedge r = FinAdmin) \vee \\ (u = u_3 \wedge r = FinClerk) \vee \\ (u = u_4 \wedge r = POAdmin) \vee \\ (u = u_5 \wedge r = PO-Clerk) \end{array} \right) \\ \forall r, p. (pa(r, p) \Leftrightarrow & \left(\begin{array}{l} (r = FinClerk \wedge p = p_4) \vee \\ (r = FinAdmin \wedge p = p_5) \vee \\ (r = PO-Clerk \wedge p = p_3) \vee \\ (r = POAdmin \wedge p = p_1) \end{array} \right) \end{aligned}$$

are satisfied by the interpretations of ua and pa in Fig. 3 and that $Manager \succeq FinAdmin$, $Manager \succeq POAdmin$, $FinAdmin \succeq FinClerk$, and $POAdmin \succeq PO-Clerk$ with the three formulae above for reflexivity, transitivity and antisymmetry make the interpretation of \succeq the partial order considered in Fig. 3. The state variable xcd allows us to formalize component (vii) of the RBAC4BPEL system about the authorization constraints. The idea is to use xcd to store the pair user u and action a when u has performed a so that

the authorization constraints can be formally expressed by a transition involving suitable pre-conditions on these variables. We illustrate the details on the first authorization constraint considered in § II-B, i.e. $\langle U, (apprPO, signGNR), \neq \rangle$. The corresponding transition can be formalized as follows:

$$\begin{aligned} & \exists x_1, x_2. (xcd(x_1, apprPO) \wedge x_1 \neq x_2 \wedge \\ & \forall x, y. (xcd'(x, y) \Leftrightarrow ((x = x_2 \wedge y = signGNR) \vee xcd(x, y)))) \end{aligned}$$

The guard of the transition prescribes that the user x_2 is not the same user x_1 that has previously performed the action $apprPO$ and the update stores in xcd the new pair $(x_2, signGNR)$. The following two constraints at the end of § II-B, namely $\langle U, (apprPO, ctrSignGNR), \neq \rangle$ and $\langle U, (signGNR, ctrSignGNR), \neq \rangle$, are formalized in a similar way. The encoding of the last constraint, i.e. $\langle R, (ctrPay, apprPay), < \rangle$, is more complex and requires also the use of the user-role relation ua to represent the constraint on the role hierarchy:

$$\begin{aligned} & \exists x_1, x_2, r_1, r_2. (xcd(x_1, crtPay) \wedge ua(x_1, r_1) \wedge \\ & ua(x_2, r_2) \wedge r_2 \succeq r_1 \wedge r_1 \neq r_2 \wedge \\ & \forall x, y. (xcd'(x, y) \Leftrightarrow ((x = x_2 \wedge y = apprPay) \vee xcd(x, y)))) \end{aligned}$$

The reader should now be convinced that every RBAC4BPEL specification can be translated into a RBAC4BPEL system.

Now, we show that the three sufficient conditions to mechanize the solution of the goal reachability problem (see § III-B) are satisfied by RBAC4BPEL systems when using forward reachability. First, the class of formulae is closed under post-image computation.

Fact 2: $Post(K, \tau_i)$ is equivalent to

$$\begin{aligned} & (\exists \underline{u}. (K(xcd) \wedge xcd(u_j, t) \wedge \xi(\underline{u}, xcd))) \vee \\ & (\exists \underline{u}. (K[\lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))] \wedge \\ & \xi[\underline{u}, \lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))])) \end{aligned}$$

where $K[\lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))]$ is the formula obtained from K by substituting each occurrence of xcd' with the λ -expression in the square brackets and then performing the β -reduction and similarly for $\xi[\underline{u}, \lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))]$. ■

As anticipated above when introducing the definition of post-image for two-level transition systems, we can eliminate the second-order quantifier over the predicate symbol xcd . Now, recall that a formula is in the *Bernays-Schönfinkel-Ramsey (BSR)* class if it has the form $\exists \underline{z} \forall \underline{w}. \phi(\underline{z}, \underline{w})$, for ϕ a quantifier-free formula and $\underline{z} \cap \underline{w} = \emptyset$ (see, e.g., [14]). As a corollary of Fact 2, it is immediate to see that if K is a BSR formula, then also $Post(\tau_i, K)$ is equivalent—to a formula in the BSR class. Since $In(xcd)$ is a formula in the BSR class, then all the formulae in the sequence FR^0, FR^1, \dots will also be BSR formulae. The second requirement is also fulfilled since the satisfiability of the BSR class is well-known to be decidable [14] and the formulae used to axiomatize the structures underlying the RBAC4BPEL transition systems are also in BSR. Third, it is possible to pre-compute a bound on the length of the sequence $FR^0, FR^1, \dots, FR^\ell$ of formulae, although the existential prefix grows after each computation of

the post-image when considering the formulae describing the set of forward reachable states. This is so because we consider only a finite and known set of users so that the length of the existentially quantified prefix is bounded by $n_u^k \times n$, where k is the maximal length of the existential prefixes of the transitions in the RBAC4BPEL system, n_u is the number of users, and n is the number of transitions.

Property 1: Let $\langle \underline{p}, In(\underline{p}), \{\tau_i(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$ be a RBAC4BPEL system, k the maximal length of the existential prefixes of τ_1, \dots, τ_n , and n_u be the cardinality of the set of users. Then, its symbolic execution tree is ℓ -complete for every $\ell \geq n_u^k \times n$. ■

The key idea of the proof is the observation that xcd is interpreted as a subset of the Cartesian product between the set of users and the set of actions whose cardinalities are bounded.

E. Combining VASs and RBAC4BPEL systems

We are now ready to fully specify applications that feature both the WF and the PM level. To do this, we consider two-level transition systems, called *VAS+RBAC4BPEL* systems, of the form

$$\langle \underline{x}, \underline{p}, In_V(\underline{x}) \wedge In_R(\underline{p}), \{\tau_i^V(\underline{x}, \underline{x}') \wedge \tau_i^R(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle,$$

where $\underline{x} = x_1, \dots, x_n$ for some $n \geq 1$, $\underline{p} = xcd$, $In_V(\underline{x})$ is the initial condition of a VAS, $In_R(\underline{p})$ is the initial condition of a RBAC4BPEL system, $\tau_i^V(\underline{x}, \underline{x}')$ is a transition of a VAS, $\tau_i^R(\underline{p}, \underline{p}')$ is a transition formula of a RBAC4BPEL system for $i = 1, \dots, n$. Note that for some transition, the guard ξ of $\tau_i^R(\underline{p}, \underline{p}')$ may be tautological since the operation involves no access-control policy restriction (e.g., the ‘flow split’ and ‘flow join’ of the Petri net in Fig. 2). It is natural to associate a VAS and an RBAC4BPEL system to a VAS+RBAC4BPEL system by projection, i.e. the associated VAS is $\langle \underline{x}, In_V(\underline{x}), \{\tau_i^V(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$ and the associated RBAC4BPEL system is $\langle \underline{p}, In_R(\underline{p}), \{\tau_i^R(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$. The structure underlying the VAS+RBAC4BPEL system is such that its reduct to the signature of the VAS is identical to the structure underlying the associated VAS and its reduct to the signature of the RBAC4BPEL system is identical to the structure underlying the associated RBAC4BPEL system.

We now show how it is possible to modularly compute the post-image of a VAS+RBAC4BPEL system by combining the post-images of the associated VAS and RBAC4BPEL system.

Fact 3: Let $K(\underline{x}, xcd) := K_V(\underline{x}) \wedge K_R(xcd)$. Then, $Post(K, \tau_i)$ is equivalent to

$$\begin{aligned} & K_V[x_j + 1, x_k - 1, x_l] \wedge \bigwedge_{i \in P} x_i \geq 0 \wedge \\ & ((\exists \underline{u}. (K_R(xcd) \wedge xcd(u_j, t) \wedge \xi(\underline{u}, xcd))) \vee \\ & (\exists \underline{u}. (K_R[\lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))] \wedge \\ & \xi[\underline{u}, \lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))])) \end{aligned}$$

where the same notational conventions of Facts 1 and 2 have been adopted. In other words, the post-image of a VAS+RBAC4BPEL system is obtained as the conjunction of the post-images of the associated VAS, denoted

with $Post_V(K, \tau_i) := Post(K_V, \tau_i^V)$, and the associated RBAC4BPEL system, denoted with $Post_R(K, \tau_i) := Post(K_R, \tau_i^R)$. Thus, we abbreviate the above formula as $Post_V(K, \tau_i) \wedge Post_R(K, \tau_i)$. ■

The proof of this fact is obtained by simple manipulations minimizing the scope of applicability of $\exists \underline{x}$ and $\exists xcd$, respectively, and then realizing that the proofs of Facts 1 and 2 can be re-used verbatim. Because of the modularity of post-image computation, it is possible to modularly define the set of forward reachable states and the symbolic execution trees for VAS+RBAC4BPEL systems in the obvious way. By modularity, we can easily show the following property.

Property 2: Let $PN := \langle P, T, F \rangle$ be an acyclic WF net, $\langle \underline{x}, In_V(\underline{x}), \{\tau_i^V(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$ be its associated VAS, and $\langle \underline{p}, In_R(\underline{p}), \{\tau_i^R(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$ be the RBAC4BPEL system with n_u users and k be the maximal length of the existential prefixes of $\tau_1^R, \dots, \tau_n^R$. Then, the symbolic reachability tree of the VAS+RBAC4BPEL system whose associated VAS and RBAC system are those specified above is ℓ -complete for every $\ell \geq \min(\max_{\pi \in \Pi} \{len(\pi|_T)\}, n_u^k \times |T|)$. ■

The key observation in the proof of this property is that in order to take a transition, the preconditions of the associated VAS and of the associated RBAC4BPEL system must be satisfied. Because of the modularity of the post-image, the duality between the set of forward reachable states and the formulae labeling the symbolic execution tree can be lifted to VAS+RBAC4BPEL. We are now ready to state and prove the main result of this paper.

Theorem 1: Let $PN := \langle P, T, F \rangle$ be an acyclic WF net and let $\langle \underline{x}, In_V(\underline{x}), \{\tau_i^V(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$ be its associated VAS. Further, let $\langle \underline{p}, In_R(\underline{p}), \{\tau_i^R(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$ be an RBAC4BPEL system with a bounded number of users. Then, the symbolic reachability problem of the VAS+RBAC4BPEL system (whose associated VAS and RBAC4BPEL system are those specified above) is decidable. ■

To mechanize this result, we can use off-the-shelf a state-of-the-art Satisfiability Modulo Theories solver such as Z3 [15] that are capable of automatically discharging the proof obligation generated by the iterated computation of the post-image in the structures underlying the VAS+RBAC4BPEL system.

To illustrate the kind of formulae arising in the application of Theorem 1, we consider the example specified in Fig. 2. In this case, we can restrict to consider three paths (projected over the transitions) in the WF net: crtPO, apprPO, ‘flow split’, signGRN, ctrSignGNR, crtPay, ‘flow join’, apprPay; crtPO, apprPO, ‘flow split’, signGRN, crtPay, crtSignGRN, ‘flow join’, apprPay; and crtPO, apprPO, ‘flow split’, crtPay, signGRN, ctrsignGRN, ‘flow join’, apprPay; each one of length eight. It is easy to see that only the first path is to be considered as the other two produce states that are equivalent since it does not matter at what time crtPay is executed with respect to signGRN and ctrSignGNR (it is possible to mechanize also this check but we leave out the details for lack of space). So, for example, it is possible to check the so-called soundness of workflows [16], i.e. to check whether

it is possible to terminate without “garbage” left. In terms of a WF net, this means that no tokens are left in places other than the special final place o of the net. This can be checked by computing the post-images of the initial state of the VAS+RBAC4BPEL system of our motivating example along the lines of Facts 1, 2, and 3 and put this in conjunction with the formula characterizing the “no-garbage” condition, i.e.

$$x_{10} \geq 1 \wedge \bigwedge_{i=1}^9 x_i = 0.$$

The resulting proof obligation, because of the closure under post-image computation of the VAS and the RBAC4BPEL system as well as the modularity of the post-image computation for the VAS+RBAC4BPEL system, is decidable as it can be put in the form $\varphi_V \wedge \varphi_R$ where φ_V is a formula of LA (whose satisfiability is decidable) and φ_R is a BSR formula (whose satisfiability is again decidable), and thus the satisfiability of their conjunction is also decidable.

IV. ANALYSIS OF AN INDUSTRIAL CASE STUDY

We have implemented a prototype tool, called *WSSMT*, that allows the user to explore the symbolic execution tree of a VAS+RBAC4BPEL system. WSSMT features a client-server architecture where the server is the Z3 SMT solver while the client (implemented in Java as an Eclipse plug-in) takes a two-level transition system and generates the proof obligations for solving the reachability problem as discussed in Theorem 1.

We have first applied WSSMT on the example described in §II to validate our ideas and then we have considered a more significant example, inspired by an industrial case study, i.e. the Digital Contract Signing (DCS, for short). The scenario consists of two signers having secure access to a trusted third party, called a Business Portal (BP), in order to digitally sign a contract. To achieve this goal, each signer sets the contract’s conditions by communicating them to BP, which creates a digital version of the contract, stores it, and coordinates the two signers in order to obtain their signatures. The DCS process is successful when both signers provide genuine signatures for the digital contract and the BP can permanently store the signed copy of the contract.

The WF level specification of the DCS consists of four BPEL processes: one for the BP, one for the two instances of the signers, one for the service checking the signature, and one for the service archiving the contract. To create the composed BPEL process out of the four components, we have used the BPEL2oWFN tool [5] that is also capable of generating a Petri net representation of the resulting process. We have modified the tool in order to generate the associated VAS as described in §III-C. As a result, we have obtained a VAS with 50 integer variables and 26 transitions.

The PM level specification of the DCS has been manually specified as there seem to be no available tool for mechanizing this task. More precisely, we have specified an RBAC4BPEL system along the lines of §III-D. The set U of users is composed of five users: two signers, the BP, one checking the signature, and one archiving the contract; the set R of

roles contains four roles corresponding to each BPEL process; the set P of permissions lists 24 elements corresponding to the right of executing the 26 transitions (2 transitions do not need authorization constraints because they are ‘flow split’ and ‘flow join’ as in the Petri net in Fig. 2 and are thus used only for synchronization at the WF level); the relation ua prescribes the obvious associations between users and roles (e.g., the two users willing to sign the contract belong to the role of signers); and the relation pa also associates the 24 permissions to the 24 transitions that need authorization constraints. Finally, we have added SoD (e.g., the user signing the contract should not be the same as the one checking the validity of the signature on the contract) and BoD (e.g., the users signing the contract should be same that have agreed on the conditions of the contract) authorization constraints.

The property that we would like to check for the DCS is that once a signed contract has been permanently stored, its signatures have been checked valid and belong to the users who provided the conditions in the contract. Indeed, to be formalized and then verified, this property requires the specification of the manipulation on the data (mainly, the contract) exchanged by the various BPEL processes. As we already observed, this is difficult if not impossible for tools like BPEL2oWFN as they consider only the control flow. One of the main advantages of using (fragments of) FOL as done in this paper is the flexibility of adding features to an available specification so as to refine it and to allow for the verification of more complex properties such as the one mentioned above. As a consequence, we have manually added to the available specification of the DCS a description of the messages exchanged among the various processes and how they are generated or modified by using well-known techniques for the specification of message-passing systems in FOL (see, e.g., [17]). For example, we were able to characterize the BPEL notion of ‘correlation set’, i.e. messages passed around contain key fields (e.g., user IDs or any business-application-specific identifiers) that can be correlated for the lifetime of the exchange and, e.g., enabled the BP to distinguish the messages sent by the first signer from those of the second. This kind of information, which is crucial for proving properties of the kind specified above, is lost while generating the Petri net by using tools such as BPEL2oWFN.

As a result, with our refined model of the DCS, we were able to verify the property about the contracts that are permanently stored given above in less than 10 seconds on a standard laptop. This is an encouraging result about the scalability of our techniques.

V. CONCLUSION

We have described automated analysis techniques for the validation of a class of web services specified in BPEL and RBAC4BPEL. We have used decidable fragments of FOL to describe the state space of this class of services and then used the state-of-the-art SMT solver Z3 to solve their reachability problems. We have applied our techniques to the verification of a digital contract signing service by using a prototype tool.

The success in solving this verification is due to the flexibility of our specification framework that allowed us to precisely capture the interplay between the control flow, the data flow, and the access-control level of the service. As future work, we plan, for instance, to extend our decidability results to WF nets containing restricted form of loops and extensions of RBAC4BPEL with delegation.

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APPENDIX

PROOFS OF § III-C

Fact 1: $Post(K, \tau_i)$ is equivalent to $K[x_j+1, x_k-1, x_l] \wedge \bigwedge_{i \in P} x_i \geq 0$, where $K[x_j+1, x_k-1, x_l]$ denotes the formula obtained by replacing x'_j with $x_j - 1$ for $j \in U^+$, x'_k with $x_k - 1$ for $k \in U^-$, and x'_l with $x_l + 1$ for $j \in U^-$.

Proof: Preliminarily, note that if $x_j = x'_j + 1$ then $x'_j = x_j - 1$ for $j \in U^+$, and if $x_k = x'_k - 1$ then $x'_k = x_k + 1$ for $k \in U^-$. Then, observe the following simple calculations:

$$\begin{aligned}
& Post(K, \tau_i) \\
\text{[by definition of } Post] \Leftrightarrow & \exists x'_1, \dots, x'_n. (K(\underline{x}') \wedge \tau_i(\underline{x}', \underline{x})) \\
\text{[by definition of } \tau_i] \Leftrightarrow & \exists x'_1, \dots, x'_n. (K(x'_1, \dots, x'_n) \wedge \\
& \bigwedge_{i \in P} x_i \geq 0 \wedge \bigwedge_{j \in U^+} x_j = x'_j + 1 \wedge \\
& \bigwedge_{k \in U^-} x_k = x'_k - 1 \wedge \bigwedge_{l \in U^-} x_l = x'_l) \\
\text{[by replacement]} \Leftrightarrow & \exists \underline{x}'. (K[x_j + 1, x_k - 1, x_l] \wedge \\
& \bigwedge_{i \in P} x_i \geq 0 \wedge \bigwedge_{j \in U^+} x_j = x'_j + 1 \wedge \\
& \bigwedge_{k \in U^-} x_k = x'_k - 1 \wedge \bigwedge_{l \in U^-} x_l = x'_l) \\
\text{[by a property of } \exists] \Leftrightarrow & K[x_j + 1, x_k - 1, x_l] \wedge \bigwedge_{i \in P} x_i \geq 0 \wedge \\
& \exists \underline{x}'. (\bigwedge_{j \in U^+} x_j = x'_j - 1 \wedge \\
& \bigwedge_{k \in U^-} x_k = x'_k + 1 \wedge \bigwedge_{l \in U^-} x_l = x'_l) \\
\text{[by a property of } \exists] \Leftrightarrow & K[x_j + 1, x_k - 1, x_l] \wedge \bigwedge_{i \in P} x_i \geq 0.
\end{aligned}$$

This concludes the proof. ■

Lemma 1: Let $PN := \langle P, T, F \rangle$ be an acyclic workflow net and Π be the set of all its paths. Then, the set of forward reachable states of the VAS $\langle \underline{x}, In(\underline{x}), \{\tau_i(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$ associated to PN is identified by the formula $FR^\ell(\tau, In)$ for $\ell = \max_{\pi \in \Pi} \{len(\pi|_T)\}$, where $\pi|_T$ is the sequence obtained from π by forgetting each of its element in P and $len(\pi|_T)$ is the length of the sequence $\pi|_T$.

Before proving the Lemma, we instantiate the notion of symbolic execution tree (introduced in § III-B) to VASs so that we can use it in the proof of the results in this part. The *symbolic execution tree of a VAS* is a labeled tree defined as follows: (i) the root node is labeled by the formula In , (ii) a node n labeled by the formula K has $d \leq n$ sons n_1, \dots, n_d labeled by the formulae $Post(\tau_1, K), \dots, Post(\tau_d, K)$ such that $Post(\tau_j, K)$ is satisfiable in the underlying model of the VAS and the edge from n to n_j is labeled by τ_j for $j = 1, \dots, d$, (iii) a node n labeled by the formula K has no son, in which case n is a *final node*, if $Post(\tau_j, K)$ is unsatisfiable in the underlying model of the VAS, for each $j = 1, \dots, n$. A symbolic execution tree of a VAS is *0-complete* if it consists of the root node labeled by the formula In , it is $(d+1)$ -complete for $d \geq 0$ if its depth is $d+1$ and for each

node n labeled by a formula K_n at depth d , if $Post(\tau_j, K_n)$ is satisfiable, then there exists a node n' at depth $d+1$ labeled by $Post(\tau_j, K_n)$.

Property 3: Let $\langle \underline{x}, In(\underline{x}), \{\tau_i(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$ be a VAS. The disjunction of all the formulae labeling a d -complete symbolic execution tree of the VAS above is logically equivalent to $FR^\ell(\tau, In)$, where $\tau := \bigvee_{i=1}^n \tau_i$.

Proof: First of all, observe that $Post$ distributes over disjunction, i.e.

$$\begin{aligned}
& Post(\tau, K) \\
\text{[by definition of } Post] \Leftrightarrow & \exists \underline{x}'. (K(\underline{x}') \wedge \tau_i(\underline{x}', \underline{x})) \\
\text{[by definition of } \tau] \Leftrightarrow & \exists \underline{x}'. (K(\underline{x}') \wedge \bigvee_{i=1}^n \tau_i(\underline{x}', \underline{x})) \\
\text{[by property of } \wedge, \vee, \exists] \Leftrightarrow & \bigvee_{i=1}^n \exists \underline{x}'. (K(\underline{x}') \wedge \tau_i(\underline{x}', \underline{x})) \\
\text{[by definition of } Post] \Leftrightarrow & \bigvee_{i=1}^n Post(\tau_i, K).
\end{aligned}$$

Then, the property follows by a simple induction on the depth of the symbolic execution tree. ■

Interestingly, we observe that the fix-point checks are always successful for every formula $FR^{\ell'}(\tau, In)$ with $\ell' > \max_{\pi \in \Pi} \{len(\pi|_T)\}$, since $FR^{\ell'}(\tau, In) = FR^\ell(\tau, In)$ as no transition is enabled in $FR^\ell(\tau, In)$. We can rephrase this in terms of the symbolic execution tree as follows.

Property 4: Let $PN := \langle P, T, F \rangle$ be an acyclic WF net. The symbolic execution tree of the VAS associated to PN is ℓ -complete for every $\ell \geq \max_{\pi \in \Pi} \{len(\pi|_T)\}$.

Proof sketch: This is a consequence of the previous property and the observation that $FR^{\ell'}(\tau, In) = FR^\ell(\tau, In)$ for every $\ell' > \max_{\pi \in \Pi} \{len(\pi|_T)\}$. ■

Now, we establish a connection between (projections of) paths in a Petri net and (projections of) paths in a symbolic execution tree.

Property 5: Let $PN := \langle P, T, F \rangle$ be a Petri net, and Π be the set of all its paths. Then, for each $\pi \in \Pi$, there exists a path π' in the symbolic execution tree of the VAS associated to PN such that $\pi|_T = \pi'|_T$. The vice versa also holds, i.e. for each π' in the symbolic execution tree of the VAS associated to PN , there exists a path $\pi \in \Pi$ such that $\pi|_T = \pi'|_T$.

Proof sketch: This is a consequence of the previous property and the fact that sets of reachable markings of Petri nets and sets of reachable states of associated VASs are in a one-to-one correspondence. ■

Lemma 1 is a consequence of the last property above and the fact that all the paths in the net are of bounded length so that it is possible to compute the one with maximal length.

PROOFS OF § III-D

Fact 2: $Post(K, \tau_i)$ is equivalent to

$$\begin{aligned}
& (\exists \underline{u}. (K(xcd) \wedge xcd(u_j, t) \wedge \xi(\underline{u}, xcd))) \vee \\
& (\exists \underline{u}. (K[\lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))] \wedge \\
& \xi[\underline{u}, \lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))])) \vee
\end{aligned}$$

where $K[\lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))]$ is the formula obtained from K by substituting each occurrence of xcd with

the λ -expression in the square brackets and then performing the β -reduction and similarly for $\xi[\underline{u}, \lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))]$.

Proof: First of all, observe the following. Assume $xcd = \lambda x, y. ((x = u_j \wedge y = t) \vee xcd'(x, y))$. We have (a) $xcd' = xcd$ if $xcd(u_j, t)$ holds and (b) $xcd' = \lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))$ otherwise (i.e. when $\neg xcd(u_j, t)$). Now, consider the following simple transformations:

$$\begin{aligned}
& Post(K, \tau_i) \\
\Leftrightarrow & \exists xcd'. (K(xcd') \wedge \tau_i(xcd', xcd)) \\
\Leftrightarrow & \exists xcd'. (K(xcd') \wedge \exists \underline{u}. (\xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow ((x = u_j \wedge y = t) \vee xcd'(x, y))))) \\
\Leftrightarrow & \exists xcd'. (K(xcd') \wedge \\
& \exists \underline{u}. ((xcd'(u_j, t) \vee \neg xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow ((x = u_j \wedge y = t) \vee xcd'(x, y))))) \\
\Leftrightarrow & \exists xcd'. (K(xcd') \wedge \\
& (\exists \underline{u}. ((xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow ((x = u_j \wedge y = t) \vee xcd'(x, y))))) \vee \\
& (\exists \underline{u}. ((\neg xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow ((x = u_j \wedge y = t) \vee xcd'(x, y))))) \\
\Leftrightarrow & \exists xcd'. (K(xcd') \wedge \\
& (\exists \underline{u}. ((xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow xcd'(x, y))) \vee \\
& (\exists \underline{u}. ((\neg xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow ((x = u_j \wedge y = t) \vee xcd'(x, y))))) \\
\Leftrightarrow & \exists xcd'. (K(xcd') \wedge \\
& (\exists \underline{u}. ((xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow xcd'(x, y))) \vee \\
& (\exists \underline{u}. ((\neg xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd'(x, y) \Leftrightarrow (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))))) \\
\Leftrightarrow & \exists xcd'. (\\
& (\exists \underline{u}. (K(xcd') \wedge (xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd(x, y) \Leftrightarrow xcd'(x, y))) \vee \\
& (\exists \underline{u}. (K(xcd') \wedge (\neg xcd'(u_j, t)) \wedge \xi(\underline{u}, xcd') \wedge \\
& \forall x, y. (xcd'(x, y) \Leftrightarrow (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))))) \\
\Leftrightarrow & (\exists \underline{u}. (K(xcd) \wedge (xcd(u_j, t)) \wedge \xi(\underline{u}, xcd))) \vee \\
& (\exists \underline{u}. (K[\lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))] \wedge \\
& \xi[\underline{u}, \lambda x, y. (\neg(x = u_j \wedge y = t) \wedge xcd(x, y))])).
\end{aligned}$$

This concludes the proof. \blacksquare

For completeness, as done for VASs, we instantiate the notion of symbolic execution tree (introduced in § III-B) to RBAC4BPEL systems. The *symbolic execution tree of a RBAC4BPEL system* is a labeled tree defined as follows: (i) the root node is labeled by the formula In , (ii) a node n labeled by the formula K has $d \leq n$ sons n_1, \dots, n_d labeled by the formulae $Post(\tau_1, K), \dots, Post(\tau_d, K)$ such that $Post(\tau_j, K)$ is satisfiable in the underlying model of the RBAC4BPEL and the edge from n to n_j is labeled by τ_j for $j = 1, \dots, d$, (iii) a node n labeled by the formula K has no son, in which case n is a *final node*, if $Post(\tau_j, K)$ is unsatisfiable in the underlying model of the RBAC4BPEL, for each $j = 1, \dots, n$. A symbolic execution tree of a RBAC4BPEL system is 0-*complete* if it consists of the root node labeled by the formula In , it is $(d+1)$ -*complete* for $d \geq 0$ if its depth is $d+1$ and for each node n labeled by a formula K_n at depth d ,

if $Post(\tau_j, K_n)$ is satisfiable, then there exists a node n' at depth $d+1$ labeled by $Post(\tau_j, K_n)$.

Property 6: Let $\langle \underline{p}, In(\underline{p}), \{\tau_i(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$ be a RBAC4BPEL system. The disjunction of all the formulae labeling a d -complete symbolic execution tree of the RBAC4BPEL system above is logically equivalent to $FR^\ell(\tau, In)$, where $\tau := \bigvee_{i=1}^n \tau_i$.

The proof is almost identical to that of Property 3 and it is thus omitted.

PROOFS OF § III-E

Preliminarily, we modularly define the sequence of formulae characterizing sets of forward reachable states and the symbolic execution trees of VAS+RBAC4BPEL systems by re-using the associated VAS and RBAC4BPEL system.

For the formulae describing the set of forward reachable states, define the following sequence, by recursion: $FR^0(K, \tau) := K$ and $FR^{i+1}(K, \tau) := (Post_V^i(FR^i, \tau) \wedge Post_R^i(FR^i, \tau)) \vee FR^i(K, \tau)$ for $i \geq 0$, $K := K_V \wedge K_R$, K_V is a formula of LA, K_R is a BSR formula, and $\tau := \bigvee_{k=1}^n \tau_k$.

For the *symbolic execution system of a VAS+RBAC4BPEL system*, preliminarily introduce the following two labeling functions. Given a node of the symbolic execution tree for a VAS+RBAC4BPEL system, the *VAS-labeling* function returns $Post_V^i(FR^i, \tau)$ while the *RBAC4BPEL-labeling* function returns $Post_R^i(FR^i, \tau)$. Then, the *symbolic execution tree of a VAS+RBAC4BPEL system* is a (multi-)labeled tree defined as follows: (i) the root node is VAS-labeled by the formula In_V and RBAC4BPEL-labeled by the formula In_R , (ii) a node n VAS-labeled by the formula K_V and RBAC4BPEL-labeled by the formula K_R has $d \leq n$ sons n_1, \dots, n_d , each n_j is VAS-labeled by the formula $Post_V(\tau_j, K)$ such that $Post_V(\tau_j, K)$ is satisfiable in the structure underlying the associated VAS and it is RBAC4BPEL-labeled by the formula $Post_R(\tau_j, K)$ such that $Post_R(\tau_j, K)$ is satisfiable in the structure underlying the associated RBAC4BPEL system, and the edge from n to n_j is labeled by τ_j for $j = 1, \dots, d$, (iii) a node n labeled by the formula K has no son, in which case n is a *final node*, if both $Post_V(\tau_j, K)$ is unsatisfiable modulo Linear Arithmetic and $Post_R(\tau_j, K)$ is unsatisfiable modulo the BSR theory, for each $j = 1, \dots, n$. A symbolic execution tree of a VAS+RBAC4BPEL system is 0-*complete* if it consists of the root node VAS-labeled by the formula In_V and RBAC-labeled by the formula In_R , it is $(d+1)$ -*complete* for $d \geq 0$ if its depth is $d+1$ and for each node n labeled by a formula K_n at depth d , if both $Post_V(\tau_j, K_n)$ is satisfiable modulo LA and $Post_R(\tau_j, K_n)$ is satisfiable modulo the BSR theory, then there exists a node n' at depth $d+1$ VAS-labeled by $Post_V(\tau_j, K_n)$ and RBAC4BPEL-labeled by $Post_R(\tau_j, K_n)$.

Property 7: Let $\langle \underline{x}, \underline{p}, In_V(\underline{x}) \wedge In_R(\underline{p}), \{\tau_i^V(\underline{x}, \underline{x}') \wedge \tau_i^R(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$ be a VAS+RBAC system. The disjunction of the conjunction between the VAS-labeling and RBAC-labeling formulae of all the nodes in a d -complete symbolic execution tree is logically equivalent to $FR^\ell(\tau, In)$, where $\tau := \bigvee_{i=1}^n \tau_i$.

The proof is along the lines of that of Property 3 and uses the modularity of the post-image computation (see Fact 3).

We are now ready to prove the main result of the paper.

Theorem 1: Let $PN := \langle P, T, F \rangle$ be an acyclic WF net and $\langle \underline{x}, In_V(\underline{x}), \{\tau_i^V(\underline{x}, \underline{x}') \mid i = 1, \dots, n\} \rangle$ be its associated VAS. Furthermore, let $\langle \underline{p}, In_R(\underline{p}), \{\tau_i^R(\underline{p}, \underline{p}') \mid i = 1, \dots, n\} \rangle$ be the RBAC system with a finite and known set of users. Then, the symbolic reachability problem of the VAS+RBAC4BPEL system whose associated VAS and RBAC4BPEL system are those specified above is decidable.

Proof: Let $G_V(\underline{x}) \wedge G_R(\underline{p})$ be a goal formula such that G_V is a LA formula and G_R be a BSR formula. By Property 2, we know that there exists a bound $\bar{\ell} \geq 0$ such that for every $\ell \geq \bar{\ell}$, the symbolic execution tree of the VAS+RBAC4BPEL system is $\bar{\ell}$ -complete. Furthermore, by Property 7, we know that the disjunction of all VAS-labeling and RBAC4BPEL-labeling formulae is equivalent to $FR^{\bar{\ell}}(\tau, In)$. Because of the $\bar{\ell}$ -completeness of the symbolic execution tree, we know that $FR^{\bar{\ell}}(\tau, In)$ is a fix-point; hence, $FR^{\bar{\ell}}(\tau, In)$ describes the set of all forward reachable states of the VAS+RBAC4BPEL system. By induction on the length of the sequence FR^0, FR^1, \dots of formulae, it is easy to show that each FR^i is equivalent to the conjunction between a formula of Linear Arithmetic, say FR_V^i , and a BSR formula, say FR_R^i . Hence, we conclude that $FR^{\bar{\ell}}(\tau, In)$ is equivalent to $FR_V^{\bar{\ell}} \wedge FR_R^{\bar{\ell}}$. Thus, in order to solve the goal reachability problem, it is sufficient to check the satisfiability of the following formula:

$$(FR_V^{\bar{\ell}} \wedge FR_R^{\bar{\ell}}) \quad \wedge \quad (G_V(\underline{x}) \wedge G_R(\underline{p}))$$

modulo the union of LA and the BSR theory. This problem can be reduced to two separate satisfiability problems modulo a single theory, namely: (i) checking the satisfiability of

$$FR_V^{\bar{\ell}} \quad \wedge \quad G_V(\underline{x})$$

modulo Linear Arithmetic and (ii) checking the satisfiability of

$$FR_R^{\bar{\ell}} \quad \wedge \quad G_R(\underline{p})$$

modulo the BSR theory. Both of these problems are well-known to be decidable and hence the overall problem is decidable. This concludes the proof. \blacksquare